





Water Resources Modelling: Part2 - Reservoir operation
Reservoir operation

Rodrigo Proença de Oliveira
Instituto Superior Técnico – Universidade de Lisboa


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Strengthening of master curricula in water resources management for the Western Balkans HEIs and stakeholders
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Reservoir operation policies

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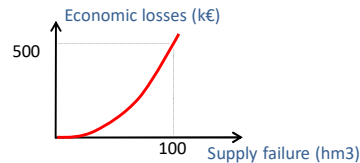
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Example

Consider a reservoir with 300 hm³ of net storage capacity and with the following inflow regime, that needs to satisfy the following water needs.

	Fall	Winter	Spring	Summer
Average inflow(hm3)	60	150	40	10
Coef. of variation of inflow	0,4	0,4	0,4	0,4
Water needs (hm3)	25	25	100	50

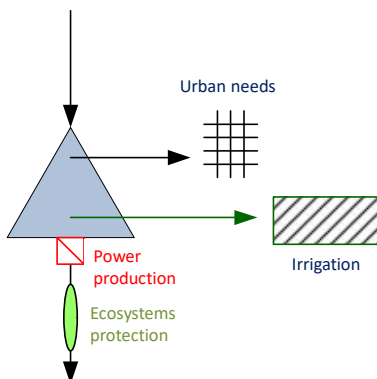
Assume that the economic losses for not providing water can be estimated by a curve such as the one on the left.



- If at the end of the Winter the water stored in reservoir is 200 hm³, how much water should the reservoir supply in the spring?
- If at the end of the Winter the water stored in reservoir is 100 hm³, how much water should the reservoir supply in the spring?

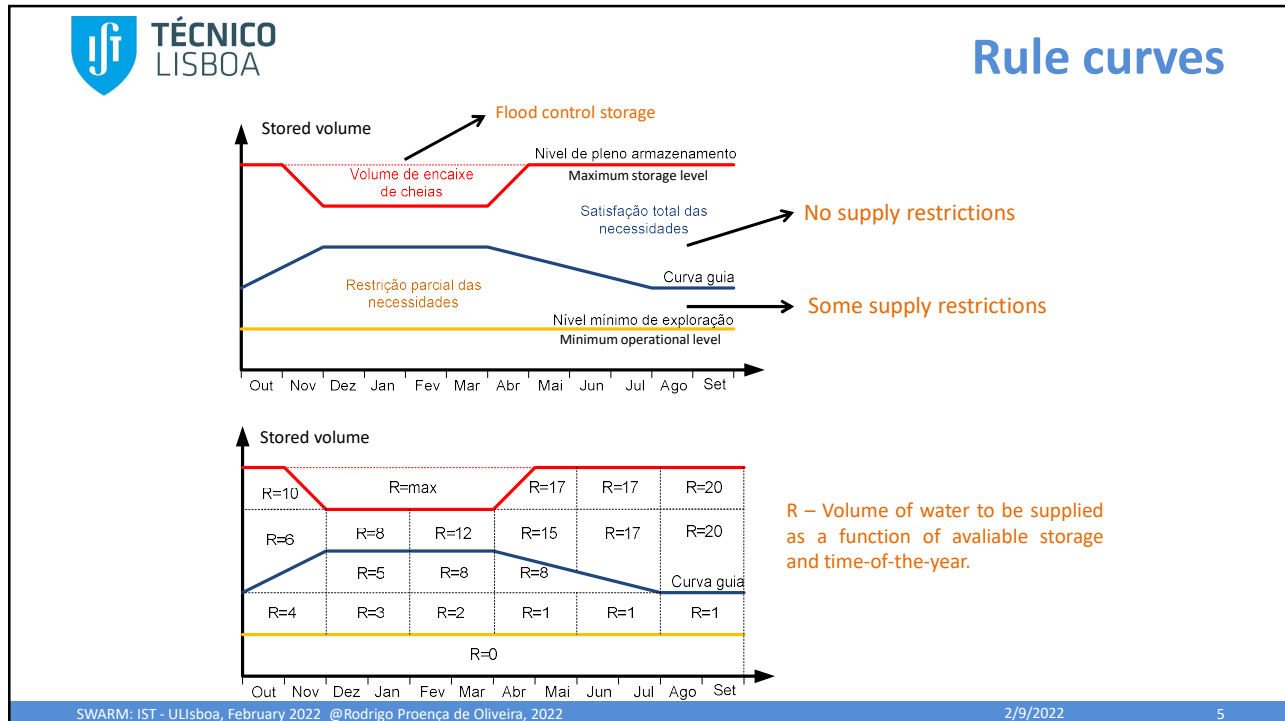
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Problem formulation

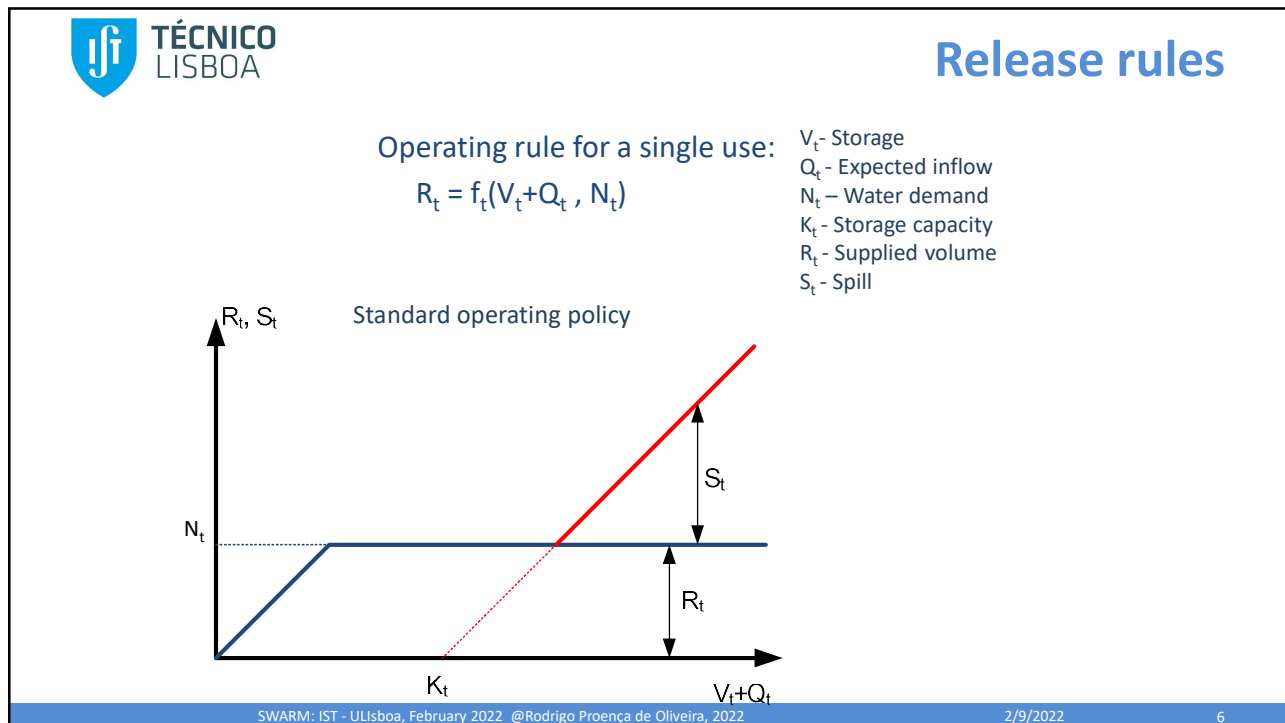


- Consider the following system which supplies water for different uses:
- How much water should we allocate to each use and how much water should we save for future use?
- What are the factors that condition the allocation of water?
 - Available water
 - Expectations on short term inflows
 - Water demands for each use and/or expected benefits for each use
- How to describe an operating policy?
 - Rule curves
 - Release rules) and balancing functions
 - Real-time mathematical models

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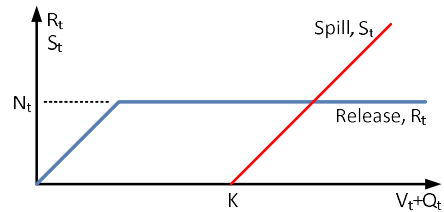
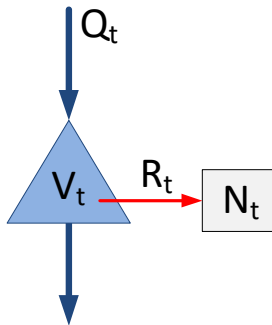
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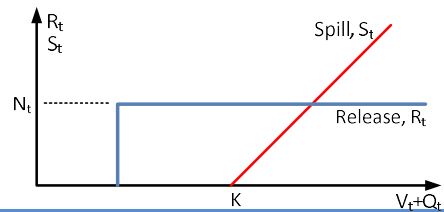
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Two possible policies, among others:

Release rule A: All available water is supplied to water needs, even it is not possible to satisfy the whole demand.

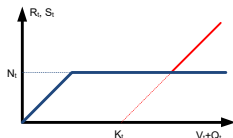


Release rule B: Water is supplied only if the whole demand is satisfied.

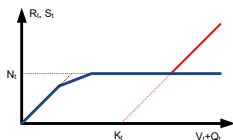


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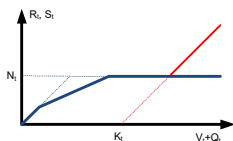
Hedging



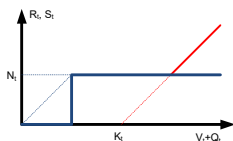
Standard operating policy



Some hedging: allows for small supply failures to safeguard future water demands; the reliability decreases

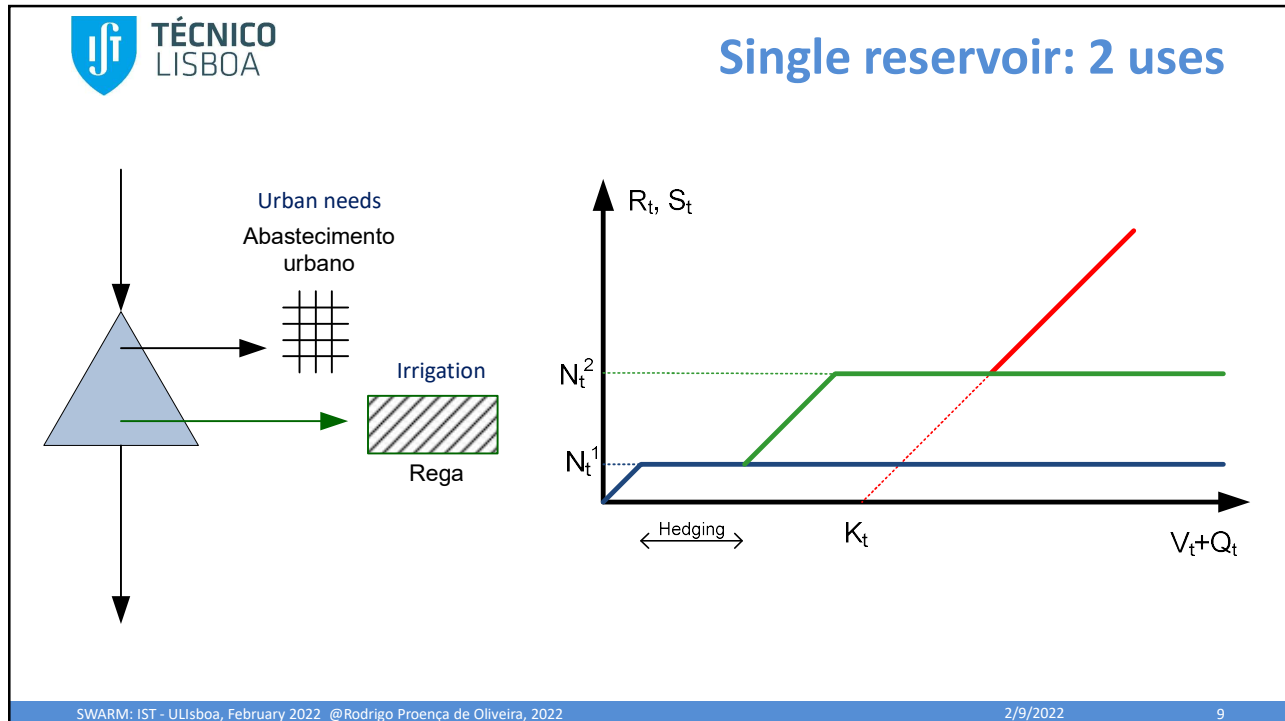


More hedging: the reliability decreases further

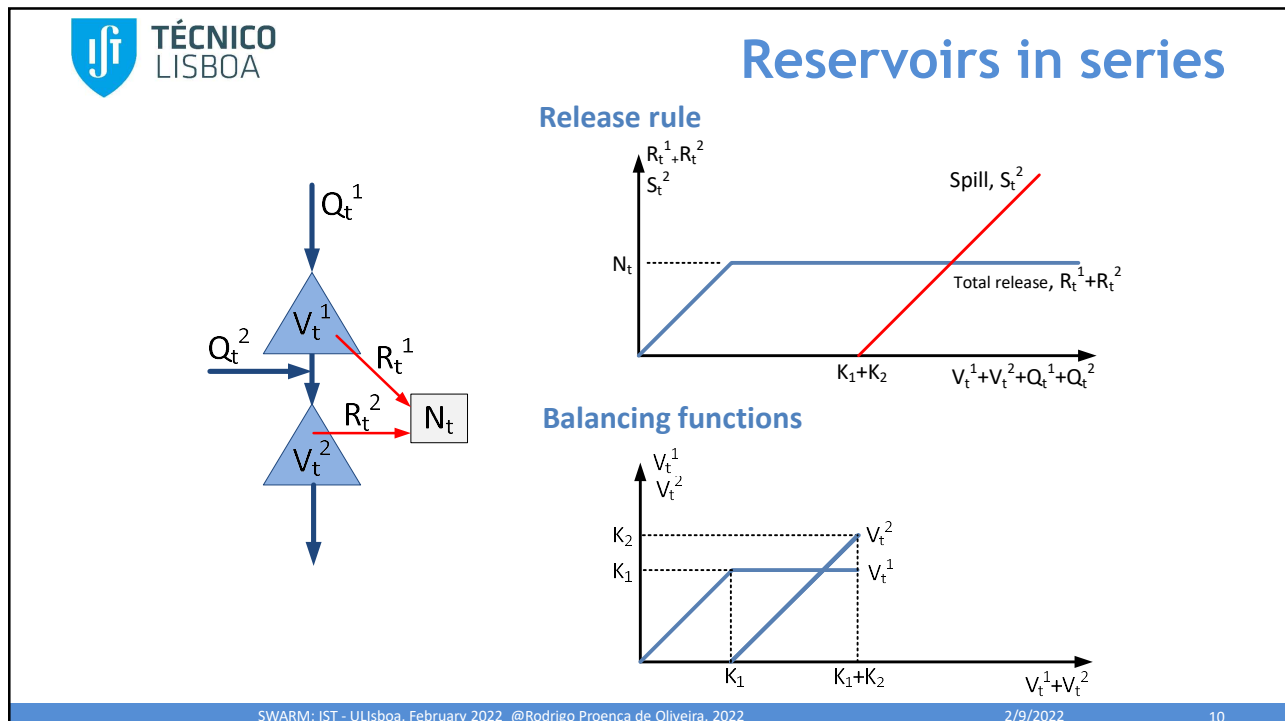


Maximizes the reliability: the number of failures is smaller but when a failure occurs no water is supplied.


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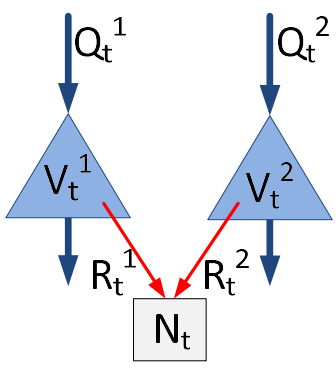
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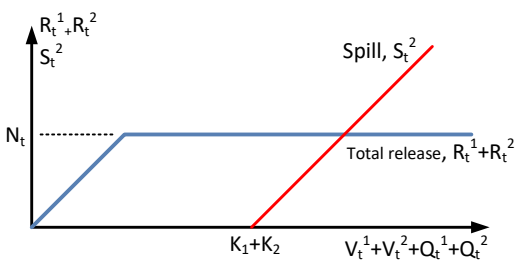


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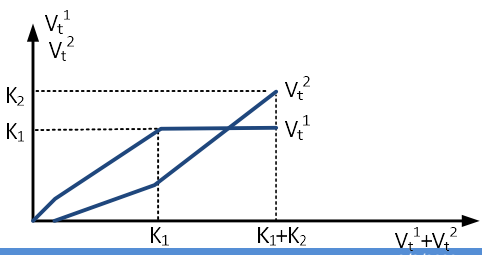
Reservoirs in parallel

Release rule






Balancing functions



$$\frac{K_1 - V_1}{Q_1} = \frac{K_2 - V_2}{Q_2}$$

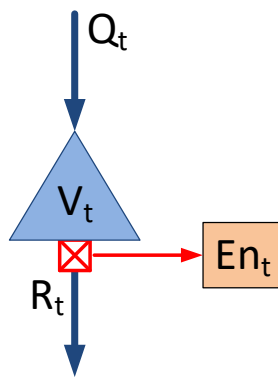
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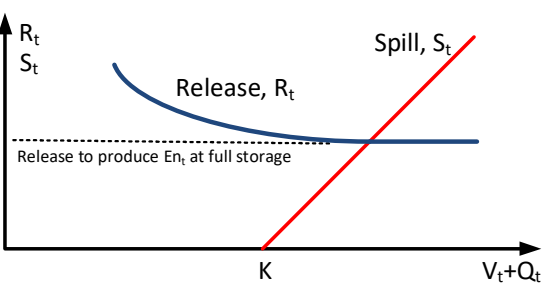


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Single reservoir for energy production



Release rule

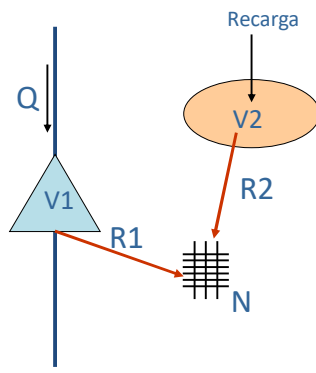


- When the stored volume is large, a large head is available for power production, which means that the amount of water needed to produce a given amount of energy is small;
- If the stored volume is small, it is not efficient to waste large amounts of water to produce a small amount of energy.

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One reservoir and one aquifer: one use

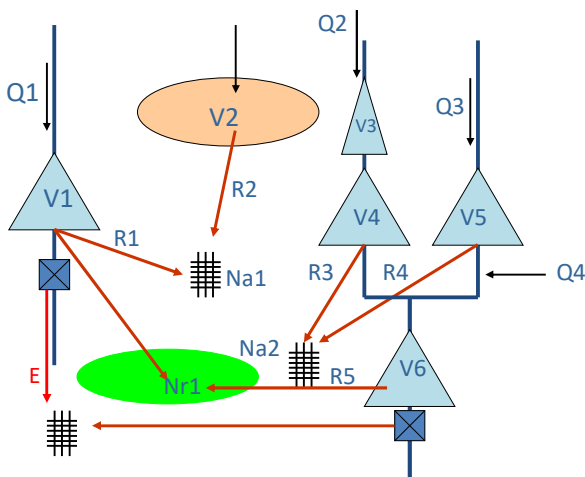


Depends on:

- Relative costs of abstraction, pumping, treatment and transport of water from both sources
- Evaporation rate (losses of water from superficial sources)
- Ratio between the reservoir capacity and the reservoir inflow (spill risk during the flooding season)
- Hydrogeological characteristics of the aquifer, namely its recharge, loss by seepage, storage capacity.

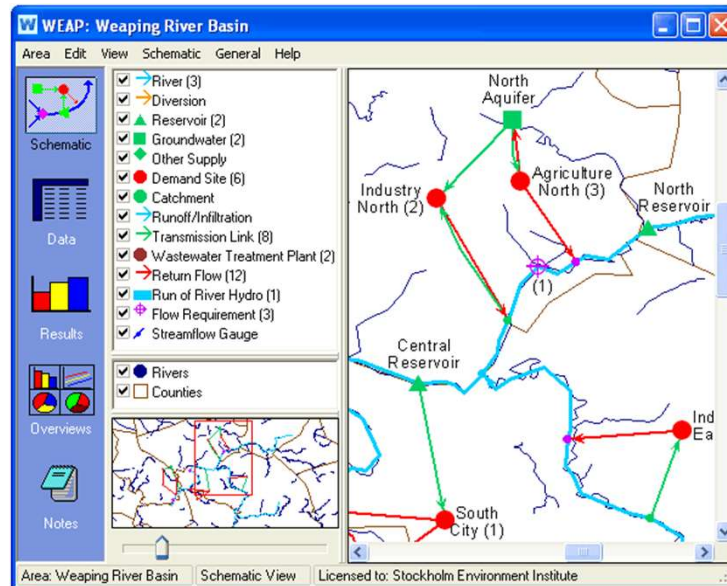
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And if ??

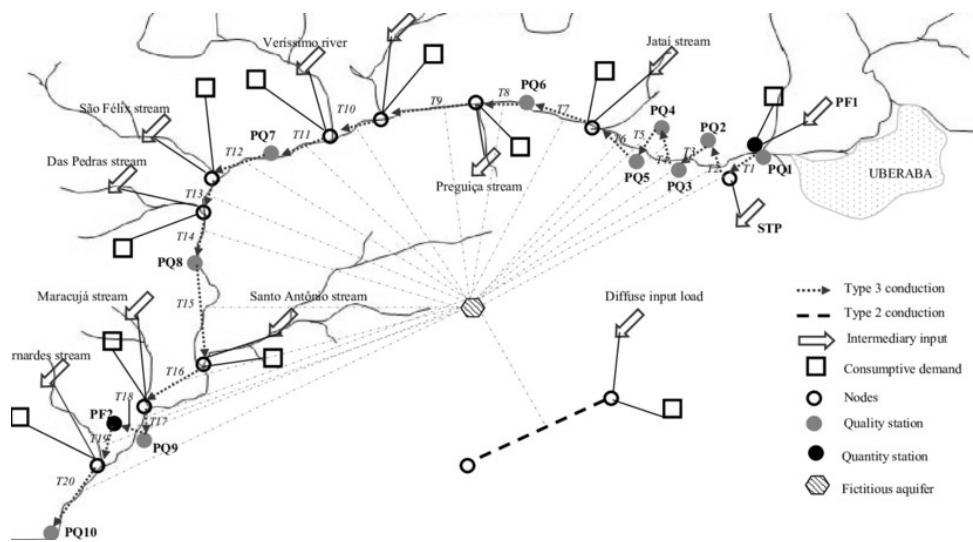


For larger and more complex systems, with a number of water sources and several users, how can we evaluate alternative management policies and select most adequate one?

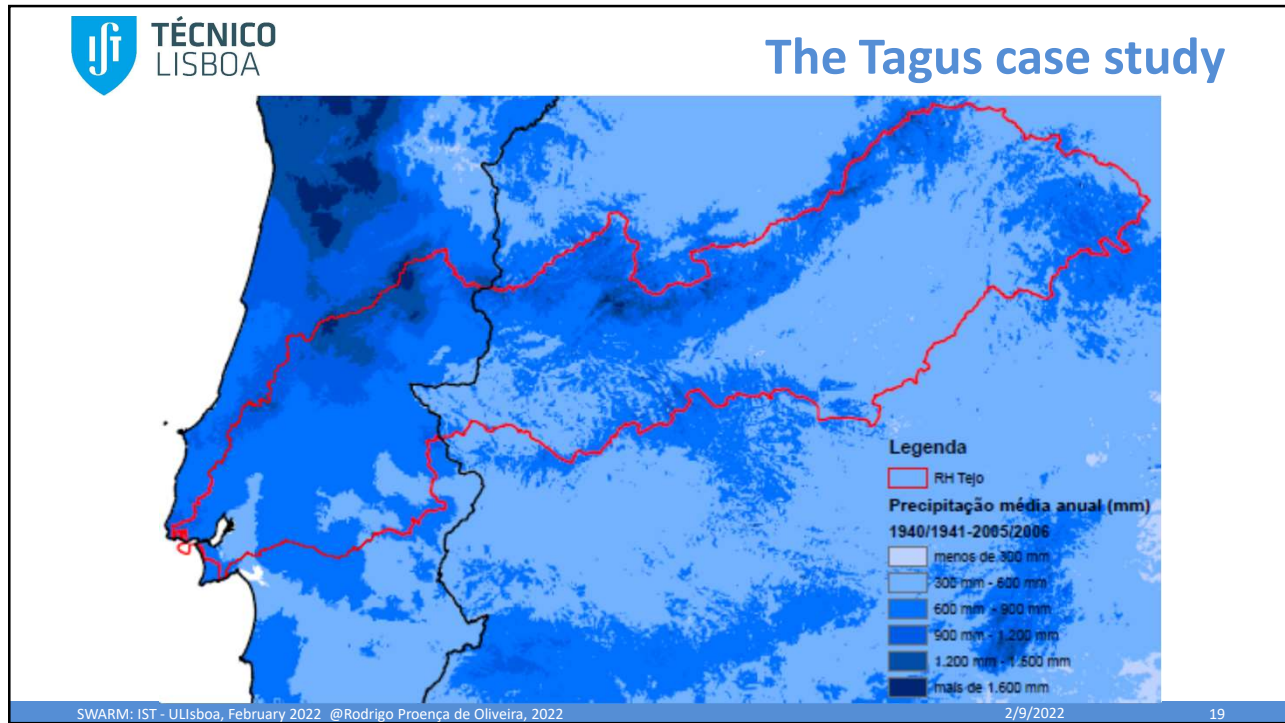
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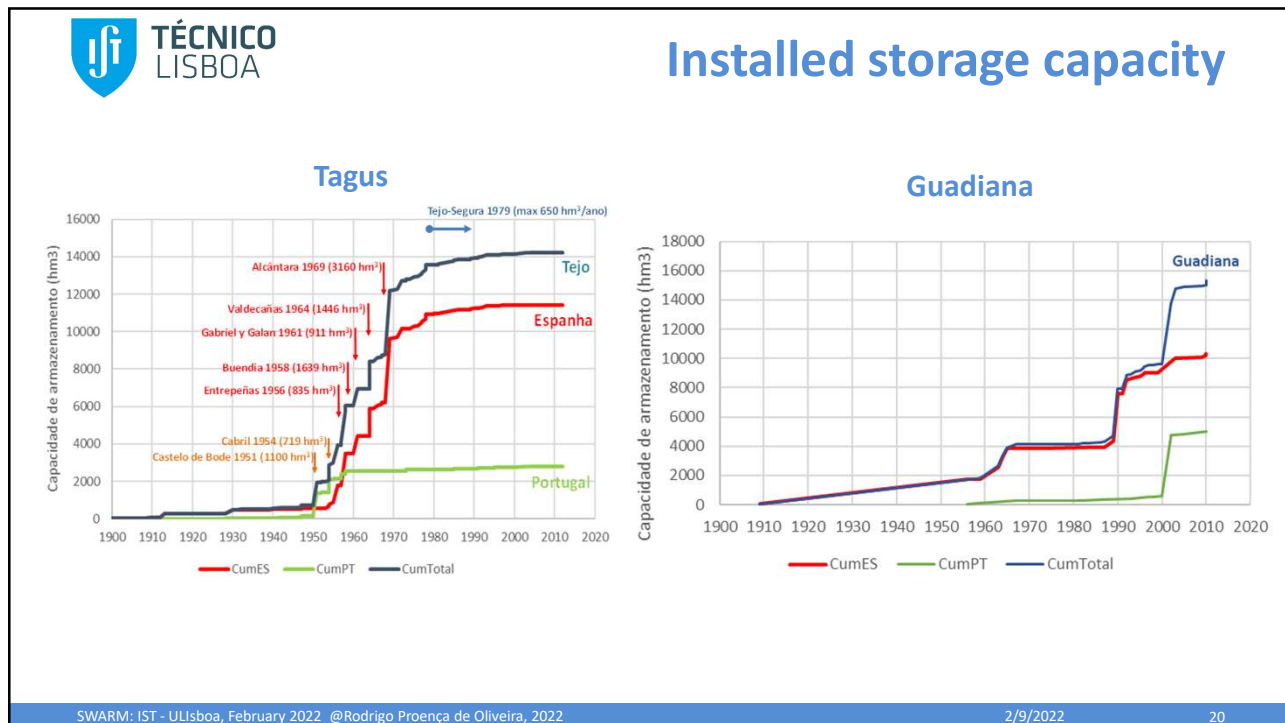
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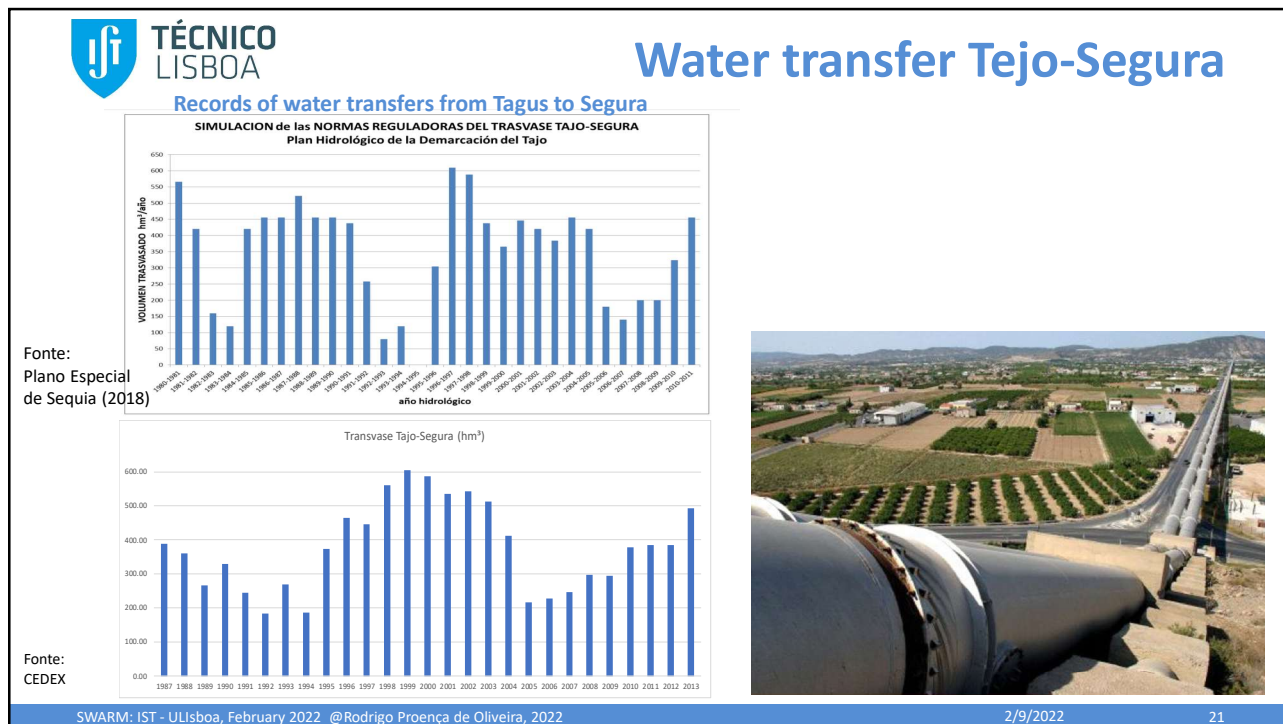
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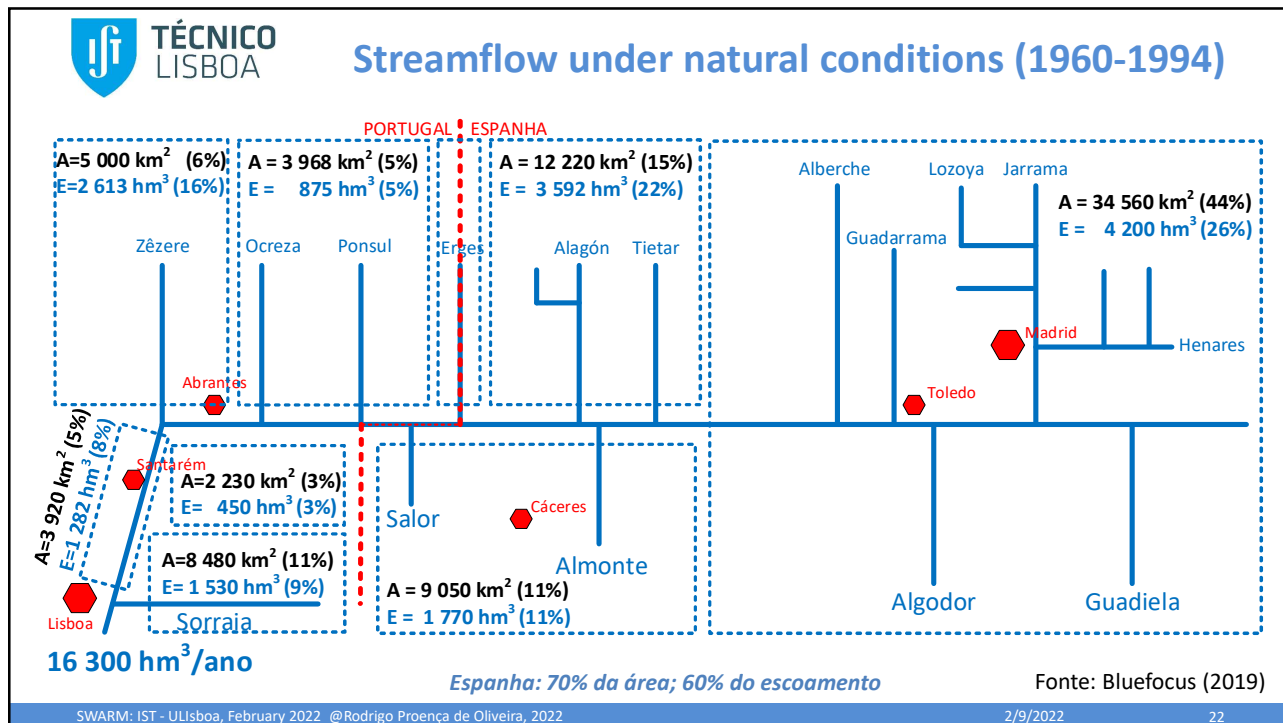
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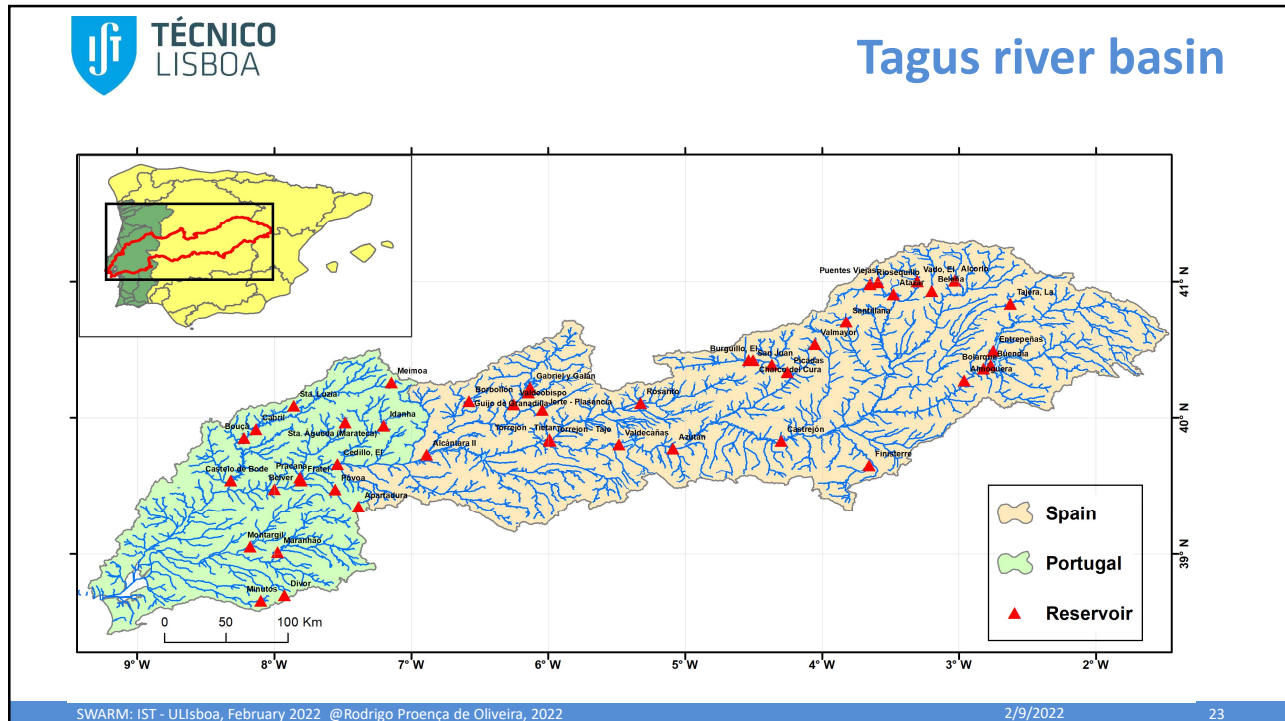
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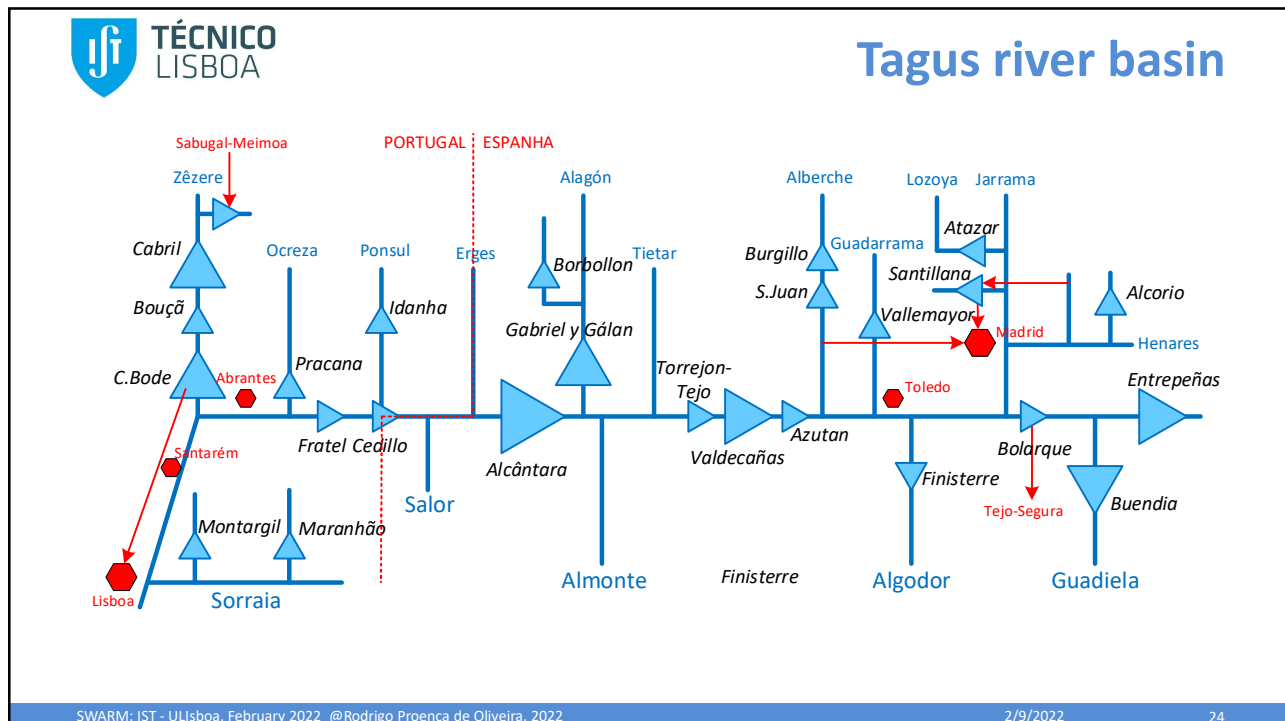
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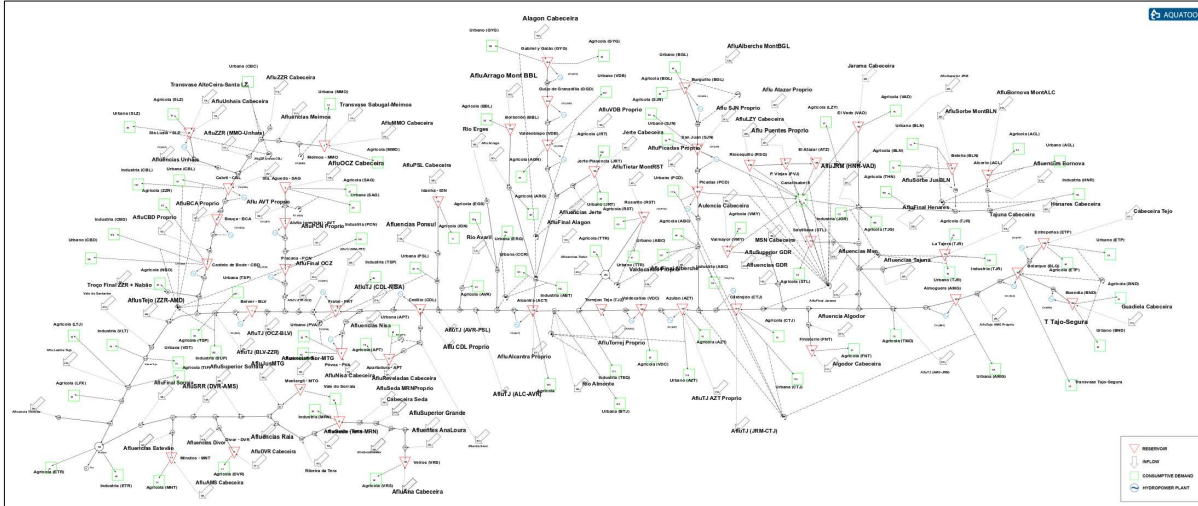
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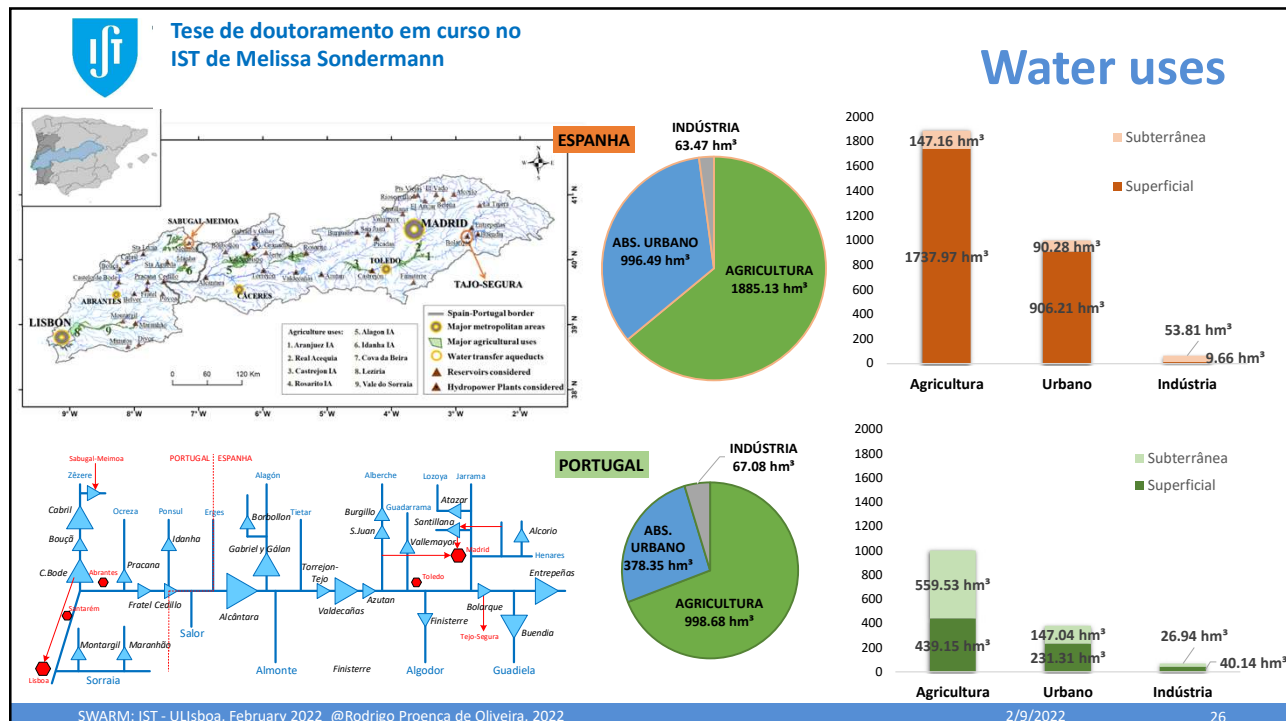


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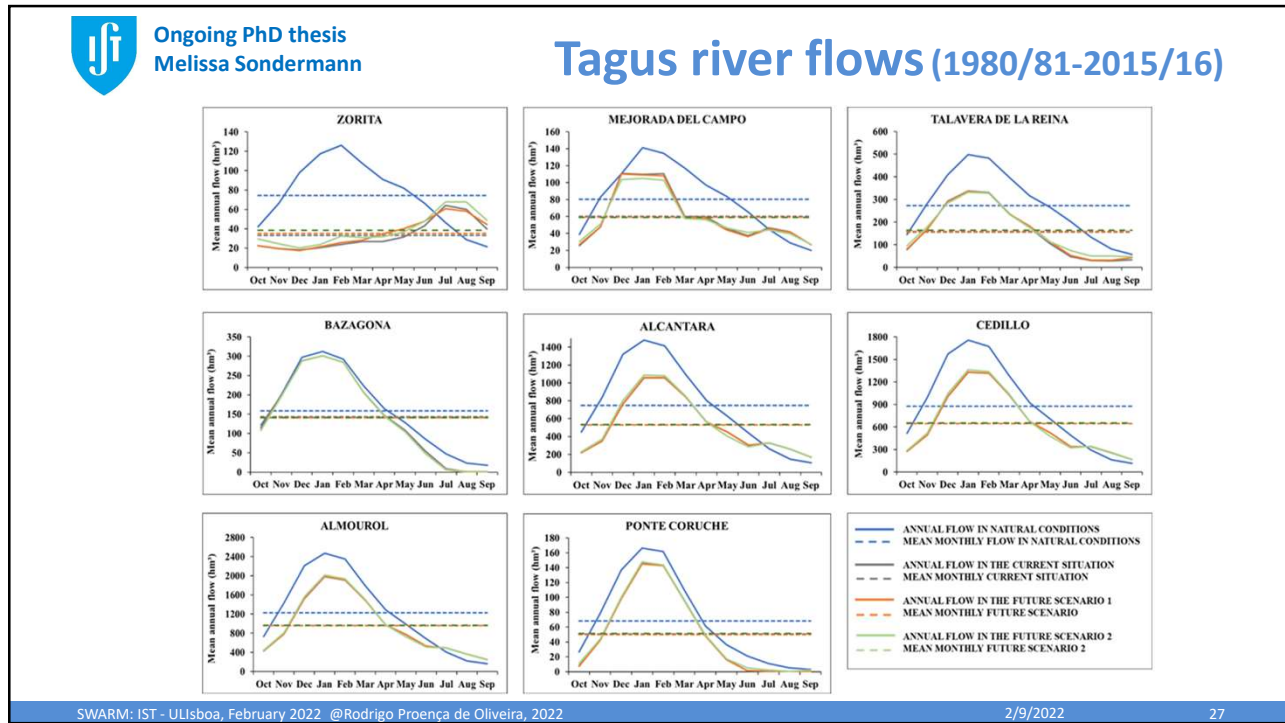


Melissa Sondermann (2019) (on going PhD thesis)

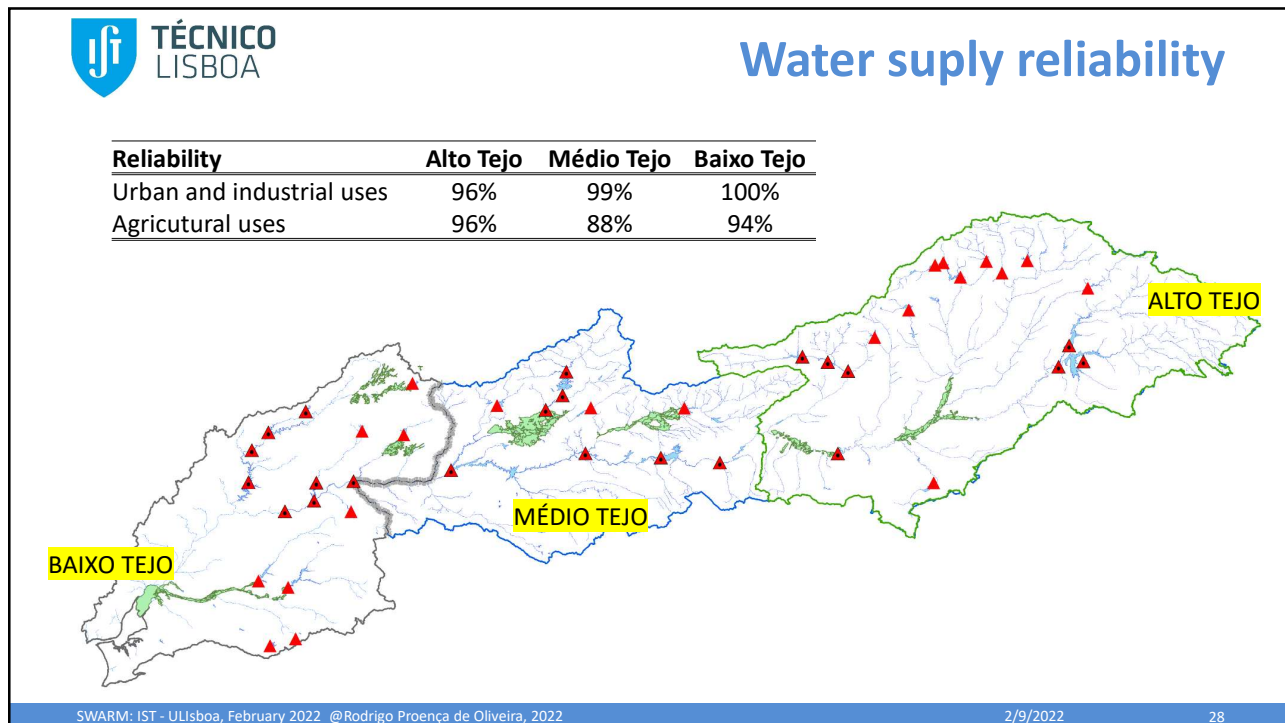
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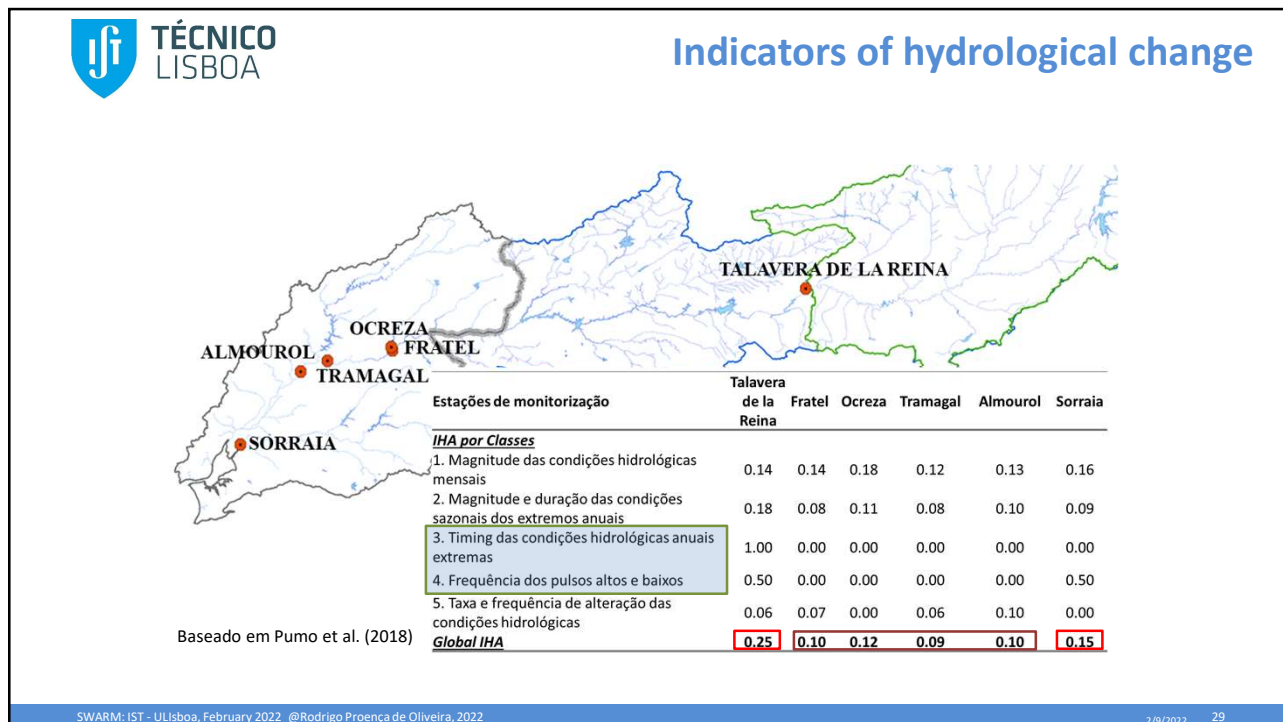
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Water Resources Modelling: Part2 - Reservoir operation

Linear programming applied to reservoir operation

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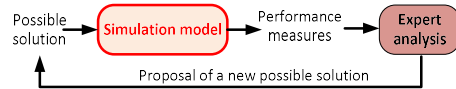
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Simulation versus optimization

Simulation

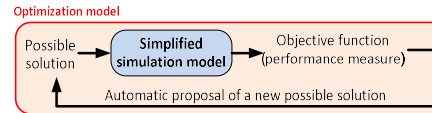


- More realistic;
- Requires manual identification of all alternatives;
- Requires manual testing of all alternatives;
- Hard work, if not impossible.

Solution:

- Use optimization for scanning solutions;
- Simulation for fine tuning.

Optimization:



- Simulation of a simplified version of the system;
- “Best” solution is identified automatically;
- The identified solution may not be best in the real world.

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Optimization problem formulation

Decision variables: define possible alternative solutions

$$X_1, X_2, \dots, X_n$$

Objective function: defines the objective to achieve (e.g. max benefits or min damages):

$$\text{Max } B(X_1, X_2, \dots, X_n)$$

Subject to restrictions: Existing conditional factors such as available water, storage capacity, etc

$$F_1(X_1, X_2, \dots, X_n) \leq b_1$$

$$F_2(X_1, X_2, \dots, X_n) \leq b_2$$

.....

$$F_n(X_1, X_2, \dots, X_n) \leq b_n$$

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Optimization techniques

- **Linear programming:** All model equations have to be linear.
- **Dynamic programming:** The objective function has to be separable / Curse of dimensionality.
- **Non-linear programming:** There are no guarantees to find the best solution.
- **Heuristic techniques:** Allow the use of simulation models.
 - Neural networks
 - Genetic algorithms
 - Simulated annealing
 - Colony optimization
- **Integer programming:** Linear Programming variant for integer variables.
- **Stochastic dynamic programming and control theory.**
- **Dynamic programming variants.**

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Linear programming

- All variables are continuous.
- All equations are linear, therefore represented straight lines, planes or hyperplanes in the solution space.
- Standard formulation:

$$\text{Max } C_1X_1 + C_2X_2 + \dots + C_nX_n$$
 Subject to:

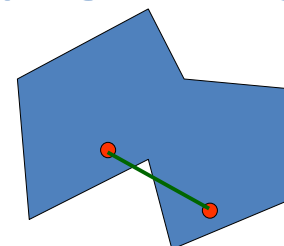
$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n \leq b_1$$

$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n \leq b_2$$

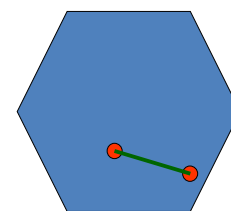
$$\dots$$

$$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n \leq b_m$$

$$X_1 \geq 0, X_2 \geq 0, \dots, X_n \geq 0$$
- The feasible solution space is a convex polygon.
- The optimal solution is always represented by a vertex of the polygon (a corner solution).
- The number of binding restrictions is equal or greater than the number of variables.



Non convex polygon

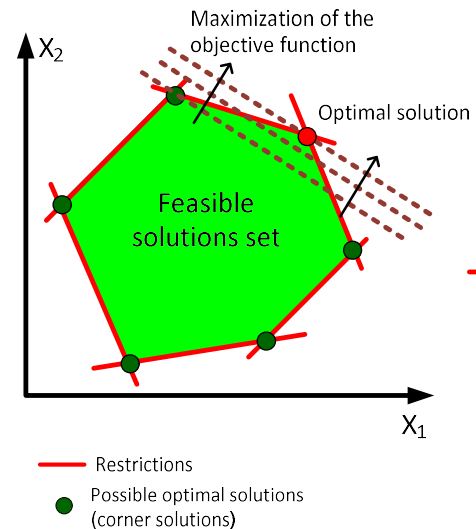


Convex polygon

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Graphical resolution

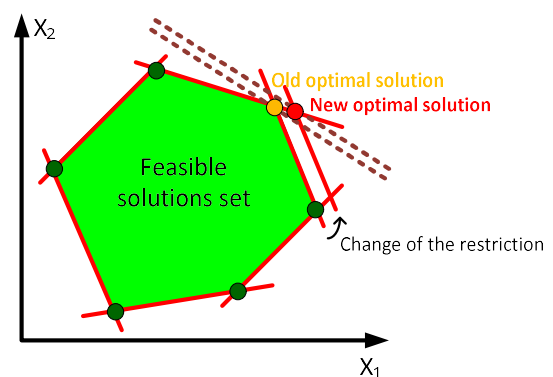
- The X and Y axis are also restrictions (non-negativity restrictions).
- Corner solutions are potential optimal solutions.
- In most situation, in a n-variable problem n restrictions condition (i.e. bind) the optimal solution .
- Binding restrictions have no slack.
- Non-binding restrictions have a positive slack.



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Shadow prices

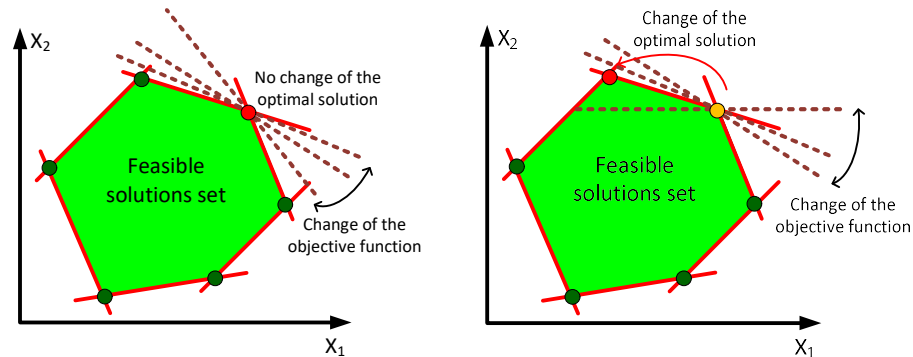
- The change of the right-hand-side of a restriction may change the optimal solution:
 $a_{i1} X_1 + a_{i2} X_2 + \dots + a_{in} X_n \leq b_i \gg$ original solution
 $a_{i1} X_1 + a_{i2} X_2 + \dots + a_{in} X_n \leq b_{i+1} \gg$ new solution
- **Shadow price** =
 New solution benefits – Original solution benefits
- The shadow price represents the cost of imposing one unit of the resource represented restriction
- The shadow price of non-binding solution is zero
- The shadow prices of non-negative restrictions are called reduced costs



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Sensitivity analysis of the objective function's coefficients

- Changes of the objective function coefficients lead to a change of the objective function “slope”.
- Small changes of the objective function coefficients do not change the optimal solution.
- Significant change may change the optimal solution.



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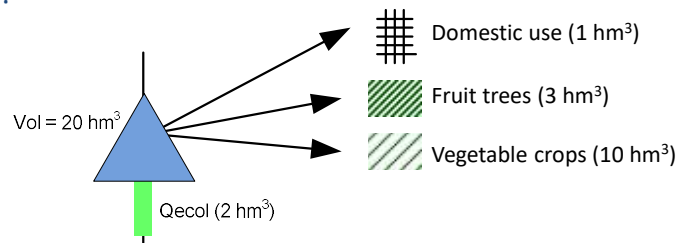
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Problem 1

At the beginning of the summer 20 hm³ of water are stored in a reservoir with a storage capacity of 30 hm³. The expected inflow in the following month is 0. The compulsory demands for the following month are 1 hm³ for domestic use and 2 hm³ for ecological needs. In addition, one should supply 3 hm³ for fruit trees e 10 hm³ for vegetable crops.

The damages for not satisfying the fruit trees and the vegetal crop water demands are 500 k€/hm³ and 80 k€/hm³, respectively. The storage at the end of the month must be greater than 5 hm³ and any volume above this level is valued at 350 k€/hm³.

How should the water be allocated?



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Problem 1: Solution

Objective function: Minimize damages

Decision variables;

P – Fruit trees water supply

H – Vegetable crops water supply

Restrictions:

Fruit trees: $P \leq 3$

Veg. Crops: $H \leq 10$

End of month volume:

$$20 + 0 - 1 - 2 - P - H \geq 5$$

$$P + H \leq 12$$

Objective function:

$$\text{Min } 500x(3-P) + 80x(10-H) - 350x(20-1-2-P-H-5) \text{ k€}$$

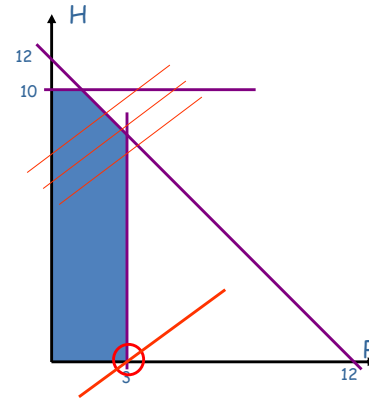
$$\text{Min } 270xH - 150xP - 1900$$

Optimal solution:

$$P = 3 \text{ hm}^3$$

$$H = 0 \text{ hm}^3$$

Damages: -2350 k euros (benefits)



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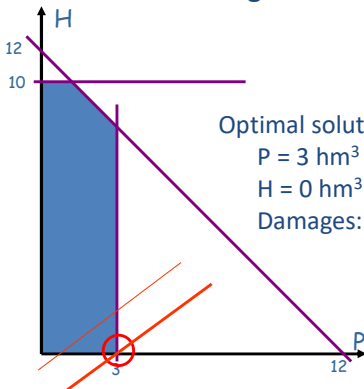
Problem 1: Further questions

Objective function: Minimize damages

Decision variables:

P – Vol. to fruit trees (hm^3)

H – Vol. to vegetable crops (hm^3)



Optimal solution:

$$P = 3 \text{ hm}^3$$

$$H = 0 \text{ hm}^3$$

Damages: -2350 Keuros

- What are the binding restrictions?
- What is the stability of the optimal solution to changes in the unit damages?
- Assuming that the value of the stored water at the end of the month is $60 \text{ k€}/\text{hm}^3$ up to 10 hm^3 and $30 \text{ k€}/\text{hm}^3$ above that value, what is new formulation of the problem?

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Problem 2

Consider a pump-storage hydropower system with a power production capacity of 1 MW and a pump capacity of 5 Mm³/month. Each Mm³ produces 30 MWh. In a specific month, there are 4 Mm³ of water to be used for power production (i.e. net water use). The market value of produced energy during peak times (10 h per weekday) is 120 €/MWh and the pumping cost during overnight is 1500 €/Mm³. If a minimum of 150 MWh must be produced during the month, how much energy should be produced during peak hours and how much water should be pumped upstream during overnight hours?

Decision variables:

R_1 - Release volume for power production during peak times (Mm³)

R_2 - Pumped volume (Mm³)

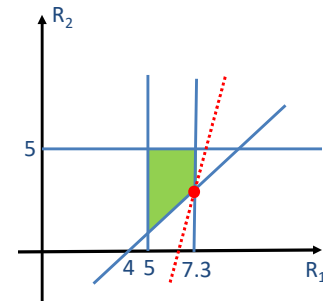
$$\text{Max } 120 \cdot 30 \cdot R_1 - 1500 \cdot R_2 \text{ euros}$$

$$R_1 \leq 4 + R_2 \text{ Mm}^3$$

$$30 \cdot R_1 \leq 1 \cdot 10 \cdot 22 \text{ MWh} \quad R_1 \leq 7.3 \text{ Mm}^3$$

$$R_2 \leq 5 \text{ Mm}^3$$

$$30 \cdot R_1 \geq 150 \text{ MWh} \quad R_1 \geq 5 \text{ Mm}^3$$



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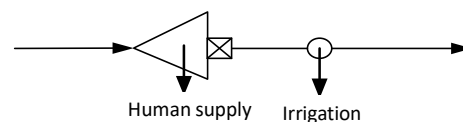
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Problem 3

In late winter, the water situation in a watershed is worrying. The stored water volume to meet the region needs amounts to 30 hm³, and the expectation for any significant water inflow are nil until next autumn. Water needs for public supply and irrigation for the spring are, respectively, 2.5 hm³ and 18 hm³. Energy needs are 1 GWh. Each hm³ of water allocated for irrigation produces 0.07 GWh.

The watershed management policy states that 2 hm³ of water for public supply needs must necessarily be met. A failure to address the remaining 0.5 hm³ causes losses amounting to 40 k€ per hm³. With respect to water requirements for irrigation it is necessary satisfy the permanent crop needs (6 hm³) and, where possible, other crop requirements (12 hm³). Irrigation needs that are not satisfied lead to losses, estimated at 20 k€ per hm³. Energy needs that area not met lead to losses, estimated at 100 k€ per GWh. Concerns regarding summer needs lead to the requirement that the volume stored at the end of the spring should be at least 16 hm³.

- What discharge decisions do you suggest to deal with this situation?
- What are the costs to satisfy the minimum requirements of urban supply and irrigation needs?
- What is the shadow price of the constraint to ensure 16 hm³ in late spring? What does this value mean?



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Problem 3: Solution

Decision variables:

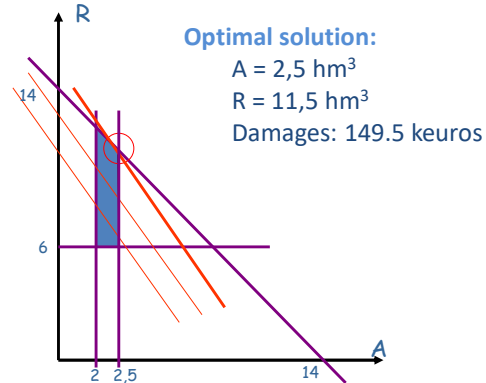
- A – Allocation for public supply (hm³)
R – Allocation for irrigation (hm³)

Restrictions:

- Public supply: $A \geq 2$
Public supply: $A \leq 2,5$
Irrigation: $R \geq 6$
Irrigation: $R \leq 18$
Summer needs: $30 + 0 - A - R \geq 16$
 $A + R \leq 14$
 $A \geq 0; R \geq 0$

Objective function

- Min $40x(2,5-A) + 20x(18-R) + 100x(1-0,07R)$
Min $560 - 40A - 27R$
Max $40A + 27R - 560$



The binding restriction is minimum volume for ensuring summer needs. We are willing to have a loss for not ensuring irrigation and energy needs.

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Problem 4

A reservoir with a capacity of 40 hm³ built for water supply, energy production, flood protection, promotion of leisure and ensuring good ecological conditions in the reservoir downstream, is in full storage at the beginning of the dry season. It is estimated that the tributary flow in the dry season is 5 hm³ and in the wet season is 15 hm³.

To ensure adequate protection against floods downstream, the reservoir has a flood control storage of 5 hm³.

As the dry season coincides with the tourist season it is crucial to ensure that the volume stored at the beginning of this season exceeds 25 hm³.

The good ecological quality in the river stretch downstream of the dam requires discharges equal to or greater than 5 hm³ in every season. Moreover, the capacity of penstock to the hydroelectric power station is 20 hm³.

The reservoir operation benefits due to water supply are 400,000 € and 200,000 € per hm³ provided respectively in the dry season and the wet season. The energy production benefits are 400,000 € and 800,000 € per hm³ provided, respectively in the dry season and the wet season. The water supply abstraction is located downstream from the dam which mean that the abstracted volumes contribute to energy production.

- What should be the reservoir operation policy?
- How much does it cost flood protection, promotion of leisure and ensuring good ecological conditions?
- What is the benefit to increase the capacity of the penstock to the hydroelectric power station?
- What is the sensitivity of the operation policy to variations in unit benefits of water supply and energy production?

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Problem 4: Solution

Decision variables:

X_1 – Release in the dry season (hm³);

X_2 – Release in the wet season (hm³).

Restrictions:

Flood protection: $40 + 5 - X_1 \leq 35$

Tourism: $40 + 5 - X_1 + 15 - X_2 \geq 25$

Reservoir capacity: $40 + 5 - X_1 + 15 - X_2 \leq 40$

Penstock capacity: $X_1 \leq 20$

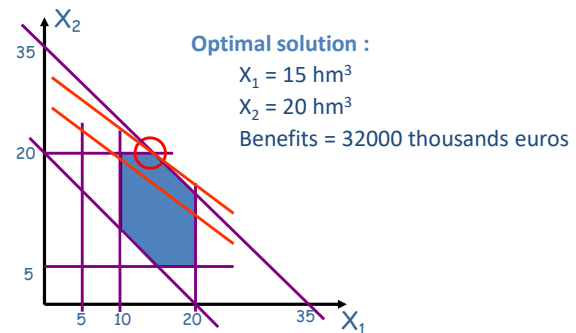
Penstock capacity: $X_2 \leq 20$

Ecological regime: $X_1 \geq 5$

Ecological regime: $X_2 \geq 5$

Objective function:

Max $(400+400)X_1 + (200+800)X_2$ (Keuros)



Shadow prices:

Flood protection: 0

Tourism: ?

Reservoir capacity: 0

Penstock capacity: ??

Penstock capacity: 0

Ecological regime: 0

Ecological regime: 0

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Linear programming with MS Excel Solver

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“Paper” solution

The problem:

Max $3X_1 + 2X_2$
 Subject to

$4X_1 + 2X_2 \leq 6$
 $4X_1 + X_2 \leq 5$
 $2X_1 + 2X_2 \leq 5$
 $X_1 \geq 0; X_2 \geq 0$

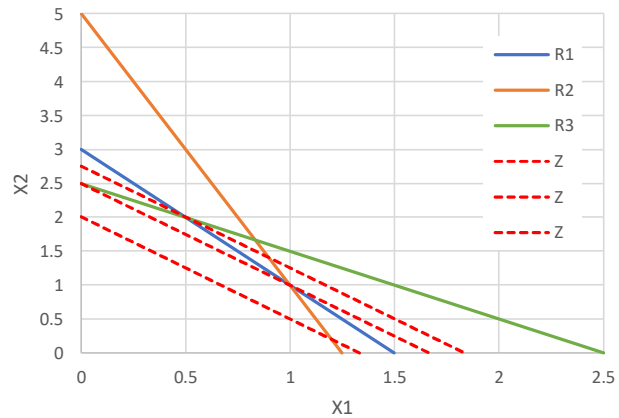
The solution:

$X_1 = 0.5$
 $X_2 = 2.0$
 $Z = 3 \times 0.5 + 2 \times 2 = 5.5$

Binding constraints: R1, R3
 Non-binding constraints: R2

Shadow prices:

R1: If $4X_1 + 2X_2 \leq 7 >$ Opt.sol.: $X_1=1; X_2=1.5; Z=6; \delta=0.5$
 R3: If $2X_1 + 2X_2 \leq 6 >$ Opt.sol.: $X_1=0; X_2=3.0; Z=6; \delta=0.5$



Sensitivity analysis of the objective function's coefficients:

Slope R3 < Slope ObjF < slope R1
 $1 < C_1/C_2 < 2$

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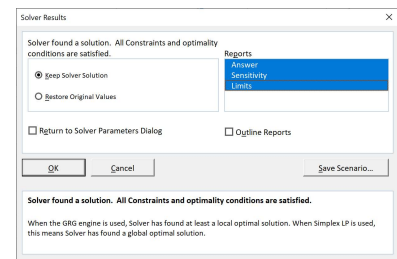
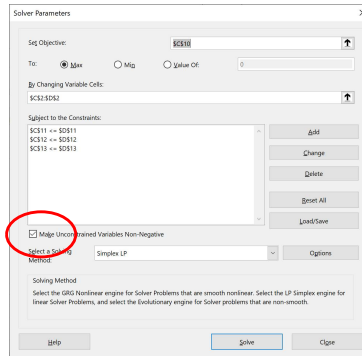
Excel solver: Setting up the problem

	A	B	C	D	
			x1	x2	
1					
2			4	5	Decision variable original values
3					
4		OF	3	2	
5		R1	4	2	
6		R2	4	1	
7		R3	2	2	
8					
9		Optimum function	LHS	RHS	
10		OF	=C4*C2+D4*D2		
11		R1	=C5*C2+D5*D2	6	
12		R2	=C6*C2+D6*D2	5	
13		R3	=C7*C2+D7*D2	5	
14					Right hand side of the restrictions
15					

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Excel-Solver: Solving the problem

- Add Solver Add in
- Solver becomes available at the Data ribbon
- Call Solver
- Insert the model in the form
- Tick linear model
- Run the model
- If Solver finds a solution keep the reports and check them; if not there is an error in the model formulation.



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Excel-Solver results

Answer report

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$C\$10	OF LHS	5	5.5

Objective function value

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$C\$2	x1	0	0.5	Contin
\$D\$2	x2	2.5	2	Contin

Decision variable values

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$C\$11	R1 LHS	6	\$C\$11<=\$D\$11	Binding	0
\$C\$12	R2 LHS	4	\$C\$12<=\$D\$12	Not Binding	1
\$C\$13	R3 LHS	5	\$C\$13<=\$D\$13	Binding	0

Info on which restriction are binding

Sensitivity analysis on the objective function coeffs.

If $C_2=2: 2 < C_1 < 4$
Original problem $C_1 = 3$

If $C_1=3: 1.5 < C_2 < 3$
Original problem $C_2 = 2$

Sensitivity report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$2	x1	0.5	0	3	1	1
\$D\$2	x2	2	0	2	1	0.5

Reduced costs: Shadow prices of the non-negative constraints

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$C\$11	R1 LHS	6	0.5	6	0.666666667	1
\$C\$12	R2 LHS	4	0	5	1E+30	1
\$C\$13	R3 LHS	5	0.5	5	1	1

Shadow prices: binding restrictions have non-zero shadow prices

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