TÉCNICO LISBOA

swarm

Co-funded by the Erasmus+ Programme of the European Union

Water Resources Modelling: Part2 - Reservoir operation

Dynamic Programming applications to reservoir operation

Rodrigo Proença de Oliveira
Instituto Superior Técnico – Universidade de Lisboa

7th – 11th February 2022

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

University of Nis

www.swarm.ni.ac.rs

Strengthening of master curricula in water resources management for the Western Balkans HEIs and stakeholders

Project number: 597888-EPP-1-2018-1-RS-EPPKA2-CBHE-JP

1

TÉCNICO LISBOA

Dynamic programming

Formulation of na optimization problem:

Each alternative solution is represented by the values the set of decision variables assume

Decision variables: $X_1, X_2, \ldots, X_n$

Objective function: Max $B(X_1, X_2, \ldots, X_n)$

subject to:

$F_1(X_1, X_2, \ldots, X_n) <= b_1$

$F_2(X_1, X2, \ldots, X_n) <= b_2$

....

$F_n(X_1, X_2, \ldots, X_n) <= b_n$

Dynamic Programming:

The objective function is a separable function, i.e. it can be defined as the sum of functions of a single variable

Max $B(X_1, X_2, \ldots, X_n) = B_1(X_1) + B_2(X_2) + \ldots + B_n(X_n)$

subject to

$F_{11}(X_1) <= b_{11}; F_{12}(X_2) <= b_{12}; \ldots; F_{1n}(X_n) <= b_{1n}$

$F_{21}(X_1) <= b_{21}; F_{12}(X_2) <= b_{22}; \ldots; F_{2n}(X_n) <= b_{2n}$

....

$F_{n1}(X_1) <= b_{n1}; F_{n2}(X_2) <= b_{n2}; \ldots; F_{1n}(X_n) <= b_{nn}$

2

TÉCNICO LISBOA

Dynamic programming

The objective function is a **separable function**, i.e. can be defined as a sum of functions of a single variables:

$$\text{Max } B(X_1, X_2, \ldots, X_n) = B_1(X_1) + B_2(X_2) + \ldots + B_n(X_n)$$

Subject to

$$F_1(X_1, X_2, \ldots, X_n) <= b_1$$

$$F_2(X_1, X_2, \ldots, X_n) <= b_2$$

....

$$F_n(X_1, X_2, \ldots, X_n) <= b_n$$

Main concepts:

- Stages: i
- Decision variables : X_i
- State variables: S_i
- Transfer function: $S_{i+1} = T(S_i, X_i)$
- Cost or benefit function: B_i
- Optimum value function

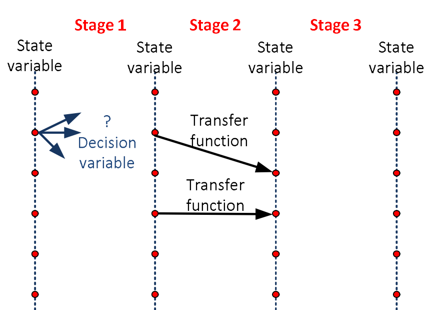
3

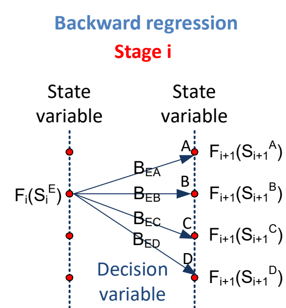
TÉCNICO LISBOA

Dynamic programming

Main concepts:

- Stages: i
- Decision variables : X_i
- State variables: S_i
- Transfer function: $S_{i+1} = T_i(S_i, X_i)$
- Cost or benefit function: $B_i(S_i, S_{i+1}, X_i)$
- Optimum value function: $F_i(S_i) = \max_{X_i}[B_i(S_i, X_i, S_{i+1}) + F_{i+1}(S_{i+1})]$





4

TÉCNICO LISBOA

Optimality principle– Belman (1957)

Belman's Principle:

- For each state and stage, there is an optimum future optimal path which is independent of how that state was reached;
- This principle allows the partition of a complex optimization problem into a set of simpler problems.

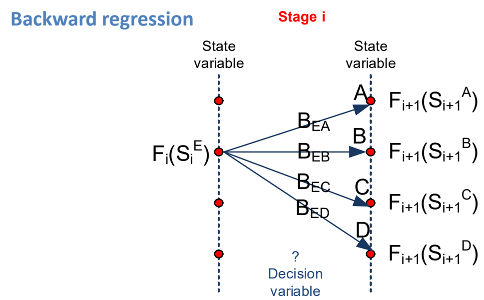
Optimal value function:

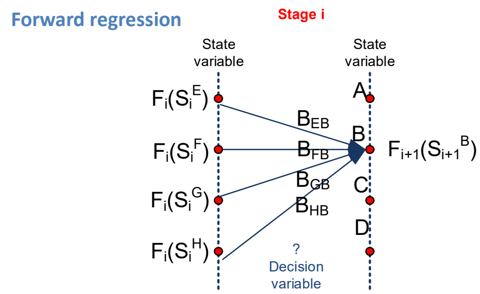
- Backward regression:

$$F_i(S_i) = \max_{X_i}[B_i(S_i, X_i, S_{i+1}) + F_{i+1}(S_{i+1})]$$

- Forward regression:

$$F_{i+1}(S_{i+1}) = \max_{X_i}[F_i(S_i) + B_i(S_i, X_i, S_{i+1})]$$





5

TÉCNICO LISBOA

Problem 1

Consider a reservoir with a net capacity of 30 and inflows equal to 10, 50 and 20, in each of the 3 seasons (4 months) of the year.

Assume that it is desirable to maintain a constant storage volume of 20 and a constant discharge of 35. The reservoir operation seeks to minimize the squared deviation of these objectives.

Determine the discharge policy for the 3 seasons and the consequent expected evolution of stored volumes, from an initial stored volume of 20.

6

TÉCNICO LISBOA

Problem 1: Solution

Stages: Seasons

State variables: Stored volumes, S_t

Decision variables: Releases, R_t

Transfer function: $S_{t+1} = S_t + Q_t - R_t$

Cost function: $C_t(S_t,R_t) = (S_t-20)^2 + (R_t-35)^2$

Optimum value function: $F(S_t) = \min \left(C_t(S_t,R_t) + F_{t+1}(S_{t+1}) \right)$

	Stage	1		
	Inflow	10		
Min F(S)	IState	Decision	FState	F(S)
375	30	10	30	1175
		20	20	475
		30	10	375
		40	0	675
475	20	0	30	1675
		10	20	775
		20	10	475
		30	0	575
875	10	0	20	1475
		10	10	975
		20	0	875
1575	0	0	10	1625
		10	0	1575

	Stage	2		
	Inflow	50		
Min F(S)	IState	Decision	FState	F(S)
450	30	50	30	450
		60	20	850
		70	10	1750
		80	0	3150
150	20	40	30	150
		50	20	350
		60	10	1050
		70	0	2250
250	10	30	30	250
		40	20	250
		50	10	750
		60	0	1750
550	0	20	30	750
		30	20	550
		40	10	850
		50	0	1650

	Stage	3					
	Inflow	20					
Min F(S)	IState	Decision	FState	F(S)		F(S)	IState
125	30	20	30	425		100	30
		30	20	125			
		40	10	225			
		50	0	725			
125	20	10	30	725		0	20
		20	20	225			
		30	10	125			
		40	0	425			
425	10	0	30	1425		100	10
		10	20	725			
		20	10	425			
		30	0	525			
1025	**0**	0	20	1625		400	**0**
		10	10	1125			
		20	0	1025			

7

TÉCNICO LISBOA

Problem 2

Consider a reservoir with a net capacity of 300 hm^3 and inflows equal to 100, 400, 300 and 0 hm^3, in each of the 4 seasons (3 months) of the year. The reservoir is equipped with a power plant and the head of the power plant is a function of the stored volume:

$h = 0.05V$.

Determine the discharge policy for the 4 seasons that maximizes the annual power production when the stored volume at beginning and end of the year is 200 hm^3.

8

TÉCNICO LISBOA

Problem 2: Solution

Stages: Seasons

State variables: Stored volumes, S_t

Decision variables: Releases, R_t

Transfer function: $S_{t+1} = S_t + Q_t - R_t$

Cost function: $B_t(S_t,R_t) = 0.05\ S_t\ R_t$

Optimum value function: $F(S_t) = \max\ (\ B_t(S_t,R_t) + F_{t+1}(S_{t+1})\ )$

Stage 1, Inflow 100

Max F(S)	IState	Decision	FState	F(S)
13500	300	100	300	13500
		200	200	12000
		300	100	11000
		400	0	13000
12000	200	0	300	12000
		100	200	10000
		200	100	8500
		300	0	10000
9000	100	0	200	9000
		100	100	7000
		200	0	8000
7000	0	0	100	6500
		100	0	7000

Stage 2, Inflow 400

Max F(S)	IState	Decision	FState	F(S)
12000	300	400	300	12000
		500	200	11000
		600	100	11000
		700	0	12000
9000	200	300	300	9000
		400	200	7500
		500	100	7000
		600	0	7500
6500	100	200	300	5500
		300	200	5000
		400	100	6000
		500	0	6500
7000	0	100	300	7000
		200	200	5500
		300	100	5000
		400	0	5500

Stage 3, Inflow 300

Max F(S)	IState	Decision	FState	F(S)
6000	300	300	300	6000
		400	200	6000
3500	200	200	300	3500
		300	200	3000
2000	100	100	300	2000
		200	200	1000
1500	0	0	300	1500
		100	200	0

Stage 4, Inflow 0

Max F(S)	IState	Decision	FState	F(S)	Max F(S)	IState (Stored volume)
1500	300	100	200	1500	0	300
0	200	0	200	0	0	200
	100				0	100
	0				0	0

TÉCNICO LISBOA

Problem 3: Reservoir operation

Consider a reservoir with a maximum storage capacity of 15 hm^3. At the beginning of a hydrological year, the reservoir stores 6 hm^3 and is expected to receive 20 hm^3 in the coming year, distributed according to the values presented in the table. The reservoir must satisfy the urban needs and the ecological flow and attempt to satisfy the irrigation needs in the spring and summer. In addition, the reservoir also needs to ensure a minimum flood protection, by reaching a storage level below 10 hm^3 at the beginning of the winter season, and to promote recreation, by ensuring a storage level above 13 hm^3 at the beginning of spring and summer. A failure to meet agriculture needs has a penalty as described in the table. The water volume stored at the end of the year is valued worth the values included in table.

Formulate and solve a dynamic programing problem to determine the best reservoir operation strategy. Clearly state the stages, the decision variables, the state variables, the transfer function and the optimal return function of this problem.

	Fall	Winter	Spring	Summer
Inflow (hm3)	6	10	4	0
Urban needs (hm3)	1	1	1	1
Irrigation needs (hm3)	0	0	4	4
Ecological flow (hm3)	2	2	2	2

Unmet agriculture demand (hm^3)	Penalty (k€)		Volume stored in the reservoir at the end of year (hm^3)	Value (k€)
0	0		>=14	1500
1	200		13	1300
2	600		12	1000
3	1200		11	500
4	2000		<=10	0

Problem 4: Reservoir operation

Consider a reservoir with a capacity of 40 hm³, a dead storage of 10 hm³ and inflows equal to 15, 20, 5 e 0 hm³ in each of the 4 seasons of the year. The reservoir serves various purposes and the overall benefits in season t can be estimated by

$B_t = 2000 - (V_{t+1} - 30)^2 - (R_t - 20)^2$,

where V_{t+1} is the volume stored in the reservoir at the end of each season and R_t the volume supplied to consumptive uses.

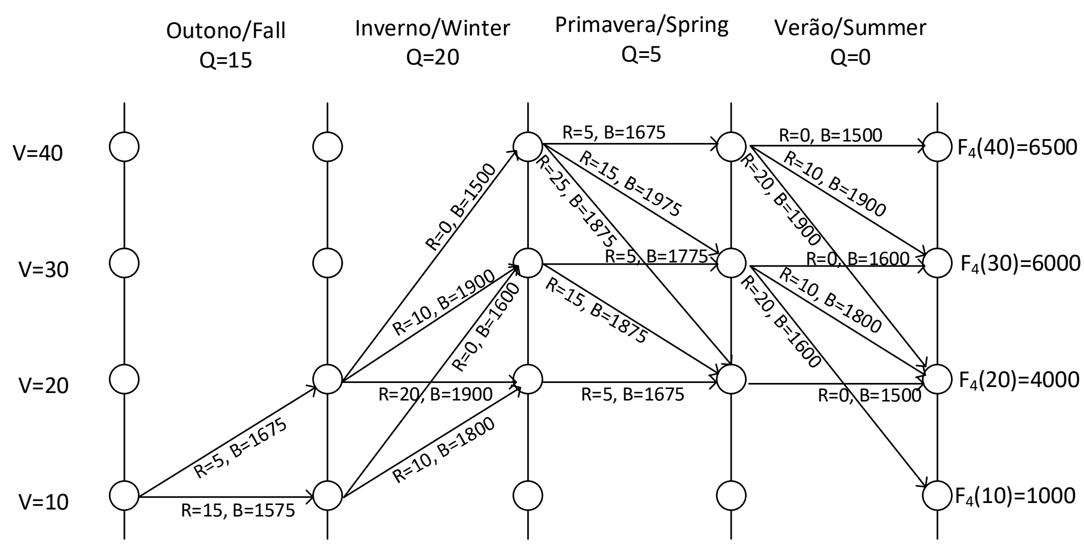
The volume at the beginning of spring has to be greater than 20 hm³ and the volume supplied to consumptive uses in each season cannot be larger than 20 hm³. The volume stored at the beginning of the year is 10 hm³.

There are benefits associated with volume stored at the end of the year, resulting from the value of stored water to meet future use. These are 6500, 6000, 4000 and 1000, respectively for a stored volume equal to 40, 30, 20 and 10 hm³.

The reservoir operation optimization problem can be solved by dynamic programming and the diagram shows the beginning of the solution. Indicate which are the decision variables, the state variables, the transfer function and the optimal return function of this problem. Complete the resolution of the problem by regressive dynamic programming and determine the solution that maximizes the benefits of long-term operation.

Explain how the procedure compares the immediate benefits in each season with the long-term benefits obtained in the following seasons. Provide a concrete example, based on the options available for the winter, when the volume stored at the beginning of the season is 20 hm³.

Problem 4: Partial solution



From this graph, we must now apply the optimum value function and identify the optimal solution.

TÉCNICO LISBOA

Problem 5

Consider a reservoir with a maximum storage capacity of 15 hm^3. At the beginning of a hydrological year, the reservoir stores 6 hm^3 and is expected to receive 20 hm^3 in the coming year, distributed according to the values presented in the table. The reservoir must satisfy the urban needs and the ecological flow and attempt to satisfy the irrigation needs in the spring and summer. In addition, the reservoir also needs to ensure a minimum flood protection, by reaching a storage level below 10 hm^3 at the beginning of the winter season, and to promote recreation, by ensuring a storage level above 13 hm^3 at the beginning of spring and summer. A failure to meet agriculture needs has a penalty as described in the table. The water volume stored at the end of the year is valued worth the values included in table.

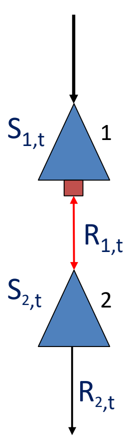
Formulate and solve a dynamic programing problem to determine the best reservoir operation strategy. Clearly state the stages, the decision variables, the state variables, the transfer function and the optimal return function of this problem.

	Fall	Winter	Spring	Summer
Inflow (hm3)	6	10	4	0
Urban needs (hm3)	1	1	1	1
Irrigation needs (hm3)	0	0	4	4
Ecological flow (hm3)	2	2	2	2

13

TÉCNICO LISBOA

Pump-storage system
(2D dynamic programming)



Stages, t: half months or half seasons

Decision variables: R_{1t}, R_{2t} (and R_{1t} may be negative)

State variables: S_{1t}, S_{2t}

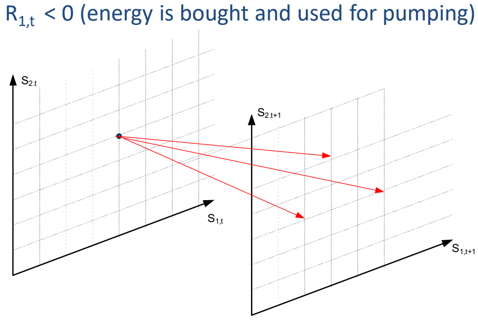
Transfer functions: $S_{1,t+1} = S_{1,t} + Q_{1,t} - R_{1,t}$

$S_{2,t+1} = S_{2,t} + R_{1,t} - R_{2,t}$

Benefit function (a, b – price of energy during the day or night)

$C = a \times S_{1,t} \times R_{1,t}$ if $R_{1,t} > 0$ (energy is produced and sold)

$C = -b \times S_{1,t} \times R_{1,t}$ if $R_{1,t} < 0$ (energy is bought and used for pumping)



For a larger number of reservoirs, there is the risk of the curse of dimensionality

14

TÉCNICO LISBOA

Problem 6: Aquifer management

Consider three pumping wells that are used to satisfy the water needs of a region, estimated as 600 m³/day. Pumping wells A, B and C can provide up to 400 m³/day, 100 m³/day and 200 m³/day, respectively. The pumping costs are shown in table. Formulate and solve a dynamic programing problem to determine the best aquifer operation strategy, clearly stating the stages, the decision variables, the state variables, the transfer function and the optimal return function of this problem.

Withdrawal (m^3/day)	Total Cost (k€/day)		
	A	B	C
0	0.10	0.15	0.05
100	0.20	0.30	0.10
200	0.40	-	0.25
300	0.65	-	-
400	0.75	-	-

Stages: Wells, i

Decision variables: Withdrawals from each well, W_i

State variables: Cumulative withdrawals up to stage i, T_i

Transfer function: $T_{i+1} = T_i + W_i$

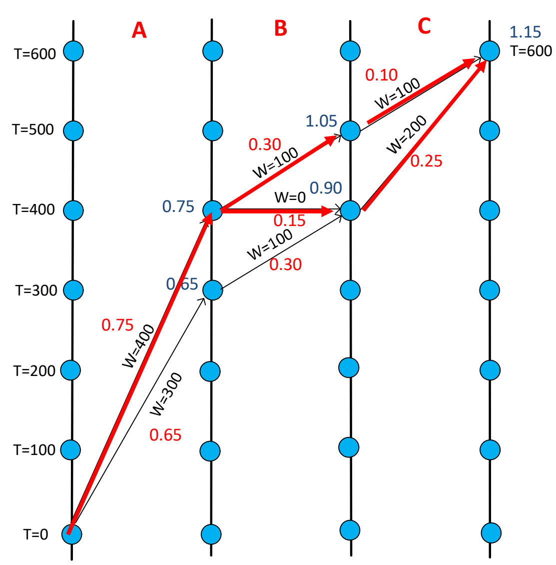
Cost function: $Cost_i = C_i(W_i)$ see table

Optimum value function: $F_{i+1}(T_{i+1}) = \min (C_i(W_i) + F_t(T_i))$

15

TÉCNICO LISBOA

Problem 6: Solution



Withdrawal (m^3/day)	Total Cost (k€/day)		
	A	B	C
0	0.10	0.15	0.05
100	0.20	0.30	0.10
200	0.40	-	0.25
300	0.65	-	-
400	0.75	-	-

From this graph, we must now apply the cost function, optimum value function and identify the optimal solution. It is easier to solve moving forward.

Optimal solution:

W_1=400 m³/day; W_2=0 m³/day; W_3=200 m³/day

Or

W_1=400 m³/day; W_2=100 m³/day; W_3=100 m³/day

Total cost: 1.15 k€/day

16

TÉCNICO LISBOA

Problem 7: Aquifer management

An aquifer lies in a region where average seasonal precipitation is as described in the following table. For each 100 mm of precipitation, the aquifer piezometric level rises 0.2 m.

	Fall	Winter	Spring	Summer
Precipitation (mm)	200	300	100	0
Piezometric level rise if no abstraction occur (m)	0,4	0,6	0,2	0,0

The aquifer is used for irrigation and the following table shows the seasonal water demand and the benefits obtained from irrigation, as a function of water supplied. The abstraction of 25 hm^3 of water from the aquifer leads to a reduction of 0,2 m of the piezometric level. It is not possible to abstract more water than is needed in a given season. Water abstraction from the aquifer has costs (e.g. pumping costs) which are shown in the following table (in thousands of euros, k€).

	Fall	Winter	Spring	Summer
Water demand (hm^3)	25	0	75	100
Volume supplied to irrigation	Benefits (k €)			
0	0	0	0	0
25	300	-	200	100
50	-	-	600	300
75	-	-	800	500
100	-	-	-	800

Aquifer piezometric level (m)	Pumping costs (k€)				
	Volume abstracted (hm^3)				
	0	25	50	75	100
10.0	0	250	400	500	600
10,2	0	200	350	400	500
10,4	0	150	300	350	400
10,6	0	100	200	300	350
10,8	0	75	150	200	300
11,0	0	50	100	150	200

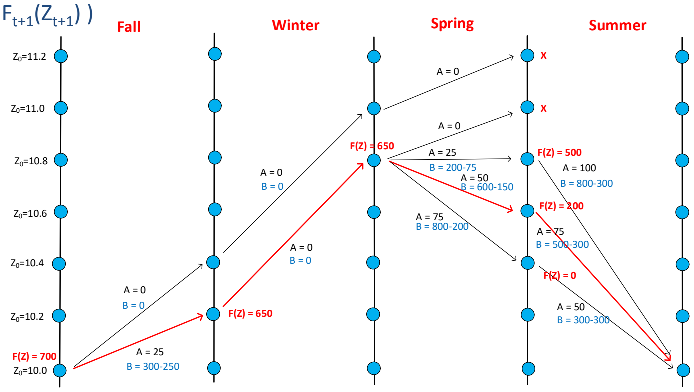
Formulate a dynamic programming problem to identify the aquifer operation policy that maximizes net benefits (benefits minus costs) over a hydrologic year, clearly identifying the state-variables, the decision variables, the objective function, the transfer function, the benefits and cost functions, and the optimal return function. At the beginning of the hydrologic year the aquifer piezometric level is at 10,0 m and the operating policy must ensure that the piezometric returns to 10,0 m at the end of the hydrologic year.

17

TÉCNICO LISBOA

Problem 7: Solution

- Stages: Seasons, t
- State variables: Piezometric level, Z_t
- Decision variables: Abstraction in each season, A_t
- Transfer function: $Z_{t+1} = Z_t + 0.2/100 \times P_t - A_t \times 0.2/25$
- Benefit function: $B_t(A_t) - C(Z_t, A_t)$
- Optimum value function: $F_t(Z_t) = \max (B_t(A_t) + F_{t+1}(Z_{t+1}))$



18

TÉCNICO LISBOA

Problem 8: Sedimentation

The channel connection to the sea from a small coastal lagoon is subject to a continuous sedimentation that reduces water exchange between the lagoon and the sea with serious consequences to the dependent aquatic ecosystem. Your firm has been hired to plan the dredging of the channel over the next year.

Studies estimate that the sedimentation in four seasons of year is, respectively, 60 cm, 100 cm, 40 cm, and 0 cm of sediment. Simultaneously, erosion removes 10% of the amount of accumulated sediment early in each station. Dredging costs can be calculated by

$$C_t = c_t \cdot {}^{D_t}/_{S_t} \ (thousands\ euros) \qquad c_t = 8{,}12{,}4{,}4 \quad for \quad t = 1{,}2{,}3{,}4$$

D_t is the amount dredged in cm and St is the cumulative amount of sediment in cm. It is not possible to dredge more than 60 cm per season. The goal is to restrict the accumulation of sediment in every season at less than 80 cm and to set the accumulation at the end of the year equal to the beginning of the year.

Formulate a dynamic programming problem that helps in the dredging plan. Clearly indicate i) the state variables, ii) the decision variables; iii) the transfer functions iv) the objective function and the cost functions v) the optimal return function, vi) and the optimal solution found. Discretize its state variables at intervals of 20 cm.

TÉCNICO LISBOA

Problem 9 – Capacity expansion

Consider an infrastructure which is requiring an increase of its capacity to deal with increasing demand. The capacity increase costs are indicated in the figure. What should be the policy for expanding the capacity of the infrastructure?